* Equilibrium of Multi product Firm:

* Multi product firm produces more than one product.

Firm
- Q_1 (Output)
- Q_2 (Output)
- Q_3 (Output)

* Such firm may produce output under condition of (a) perfect competition
(b) Under imperfect competition or monopoly.

When a firm produces two output suppose Q_1 & Q_2 jointly then total cost (TC) will be a function of Q_1 & Q_2

i.e; $TC = f(Q_1, Q_2)$

Similarly prices for both the goods will be different say $\bar{P}_1$ & $\bar{P}_2$

Hence total revenue $TR = \bar{P}_1 q_1 + \bar{P}_2 q_2$

$\therefore TR = TR_1 + TR_2$

Profit function

$$\pi = TR_1 + TR_2 - TC$$
$$= \bar{P}_1 q_1 + \bar{P}_2 q_2 - C(q_1, q_2)$$

Profit maximising conditions

$$\frac{\partial \pi}{\partial q_1} = \frac{\partial \pi}{\partial q_2} = 0 \quad \text{(1st order)}$$

and $|H_1| = \dfrac{\partial^2 \pi}{\partial q_1^2} < 0$

$$|H_2| = \begin{vmatrix} \dfrac{\partial^2 \pi}{\partial q_1^2} & \dfrac{\partial^2 \pi}{\partial q_1 \partial q_2} \\ \dfrac{\partial^2 \pi}{\partial q_2 \partial q_1} & \dfrac{\partial^2 \pi}{\partial q_2^2} \end{vmatrix} > 0$$

Q:- A monopolist produces his product in two different plants and his total cost function are given by

$$TC_1 = 20 - 4Q_1 + \frac{1}{2}Q_1^2$$
$$TC_2 = 40 - 8Q_2 + Q_2^2$$

If the average revenue function is given by $AR = 40 - Q$ where $Q = Q_1 + Q_2$. Find profit maximising output and maximum profit.

Ans: Given that $P = 40 - Q$.

$$\therefore TR = P \times Q$$
$$= (40 - Q)Q$$
$$= 40Q - Q^2$$
$$= 40(Q_1 + Q_2) - (Q_1 + Q_2)^2$$

1st order condition for maximum is

$$\frac{\partial \pi}{\partial q_1} = \frac{\partial \pi}{\partial q_2} = 0$$

$$\therefore \quad 44 - 3q_1 - 2q_2 = 0 \quad \text{—(1)}$$

$$48 - 2q_1 - 4q_2 = 0 \quad \text{—(2)}$$

eq^n (1) × 2 − eq^n (1)

$$88 - 6q_1 - 4q_2 = 0$$

$$48 - 2q_1 - 4q_2 = 0$$

$$40 - 4q_1 = 0$$

$$\Rightarrow 4q_1 = 40$$

$$\therefore q_1 = 10$$

putting the value of q_1 in eq^n (1)

$$44 - 3q_1 - 2q_2 = 0$$

$$\Rightarrow 44 - 3\times 10 - 2q_2 = 0$$

$$\Rightarrow 44 - 30 - 2q_2 = 0$$

$$\Rightarrow 14 - 2q_2 = 0$$

$$\Rightarrow 2q_2 = 14 \qquad \therefore q_2 = 7.$$

$$= 40q_1 + 40q_2 - q_1^2 - 2q_1q_2 - q_2^2$$

$$TC = TC_1 + TC_2$$

$$= 20 - 4q_1 + \tfrac{1}{2}q_1^2 + 40 - 8q_2 + q_2^2$$

$$= 60 - 4q_1 + \tfrac{1}{2}q_1^2 - 8q_2 + q_2^2$$

Profit $\pi = TR - TC$

$$= 40q_1 + 40q_2 - q_1^2 - 2q_1q_2 - q_2^2 - 60 + 4q_1 - \tfrac{1}{2}q_1^2 + 8q_2 - q_2^2$$

$$= 44q_1 + 48q_2 - \tfrac{3}{2}q_1^2 - 2q_2^2 - 2q_1q_2 - 60$$

$$\frac{\partial \pi}{\partial q_1} = 44 - 3q_1 - 2q_2$$

$$\frac{\partial \pi}{\partial q_2} = 48 - 4q_2 - 2q_1$$

$$\frac{\partial^2 \pi}{\partial q_1^2} = -3$$

$$\frac{\partial^2 \pi}{\partial q_2 \partial q_1} = -2$$

$$\frac{\partial^2 \pi}{\partial q_2^2} = -4$$

$$\frac{\partial^2 \pi}{\partial q_1 \partial q_2} = -2$$

At $q_1 = 10$ & $q_2 = 7$

$$|H_1| = \frac{\partial^2 \pi}{\partial q_1^2} = -3 < 0$$

$$|H_2| = \begin{vmatrix} \frac{\partial^2 \pi}{\partial q_1^2} & \frac{\partial^2 \pi}{\partial q_2 \partial q_1} \\ \frac{\partial^2 \pi}{\partial q_1 \partial q_2} & \frac{\partial^2 \pi}{\partial q_2^2} \end{vmatrix} = \begin{vmatrix} -3 & -2 \\ -2 & -4 \end{vmatrix}.$$

$$= 12 - 4 = 8 > 0$$

Thus profit maximizing output

$q_1 = 10$ & $q_2 = 7$.

Maximum profit $\pi = 44q_1 + 48q_2 - \frac{3}{2}q_1^2 - 2q_2^2 - 2q_1q_2 - 60$

$$= 44 \times 10 + 48 \times 7 - \frac{3}{2}(10)^2 - 2(7)^2 - 2 \times 10 \times 7 - 60$$

$$= 440 + 336 - 150 - 98 - 140 - 60$$

$$= 776 - 448$$

$$= 328$$