in § 10.3. ... will be

10.5 Energetics of nuclear reactions

During a nuclear reaction, energy is either evolved or absorbed. Reactions in which energy is evolved are known as *exoergic reactions* while those requiring absorption of energy are called *endoergic*. The total amount of energy evolved or absorbed during a nuclear reaction is called the Q value or simply the Q of the reaction. So by definition

$$Q = E_Y + E_y - E_X - E_x = E_Y + E_y - E_x \qquad ...(10.5\text{-}1)$$

if the target nucleus X is at rest.

Q is thus equal to the net surplus (or deficit) of the energies of the reaction products $E_Y + E_y$ over the energy supplied (E_x).

If the atomic masses are expressed in energy units, Eq. (10.3-3) can be rewritten as

$$M_X + M_x + E_x = M_Y + M_y + E_Y + E_y$$

Then we get from Eq. (10.5-1)

$$Q = M_X + M_x - M_Y - M_y \qquad ...(10.5\text{-}2)$$

Written in terms of the binding energies of different nuclei, we can also write

$$Q = B_Y + B_y - B_X - B_x \qquad ...(10.5\text{-}3)$$

By definition $Q > 0$ for an exoergic reaction, while $Q < 0$ for an endoergic reaction. Since there is a net deficit of energy in the latter case some energy must be supplied for the reaction to occur. This usually comes from the kinetic energy E_x of the projectile.

Eq. (10.5-2) shows that for an exoergic reaction $M_X + M_x$ is greater than $M_Y + M_y$ while for an endoergic reaction $M_X + M_x$ is less than $M_Y + M_y$.

Threshold energy of an endoergic reaction :

Eq. (10.5-1) shows that the Q value of a reaction can be expressed in terms of the kinetic energies of the projectile (E_x) and of the product nuclei E_y and E_Y.

In view of the energy and momentum conservation laws, E_Y can be expressed in terms of E_x and E_y. Refering to Fig. 10.5, we get from the law of conservation of momentum along and perpendicular to the direction of motion of the projectile ($\because p = \sqrt{2ME}$)

$$\sqrt{2M_x E_x} = \sqrt{2M_y E_y}\cos\theta + \sqrt{2M_Y E_Y}\cos\varphi \qquad ...(10.5\text{-}4)$$

$$0 = \sqrt{2M_y E_y}\sin\theta - \sqrt{2M_Y E_Y}\sin\varphi \qquad ...(10.5\text{-}5)$$

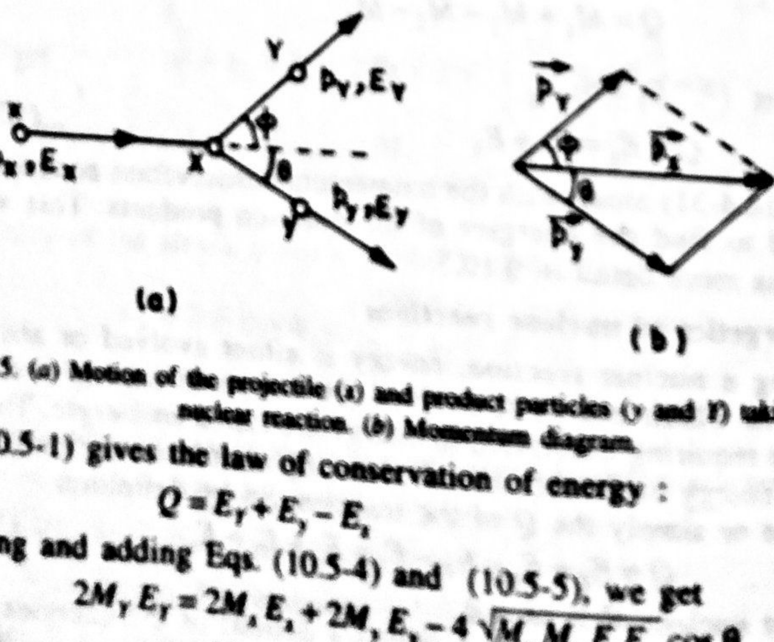

Fig. 10.5. (a) Motion of the projectile (x) and product particles (y and Y) taking part in a nuclear reaction. (b) Momentum diagram.

Eq. (10.5-1) gives the law of conservation of energy :

$$Q = E_Y + E_y - E_x$$

Squaring and adding Eqs. (10.5-4) and (10.5-5), we get

$$2M_Y E_Y = 2M_x E_x + 2M_y E_y - 4\sqrt{M_x M_y E_x E_y}\cos\theta$$

or

$$E_Y = \frac{M_x}{M_Y}E_x + \frac{M_y}{M_Y}E_y - \frac{2}{M_Y}\sqrt{M_x M_y E_x E_y}\cos\theta \qquad ...(10.5\text{-}6)$$

Then from Eq. (10.5-1) and (10.5-6) we get

$$Q = E_y\left(1 + \frac{M_y}{M_Y}\right) - E_x\left(1 - \frac{M_x}{M_Y}\right) - \frac{2}{M_Y}\sqrt{M_x M_y E_x E_y}\cos\theta \qquad ...(10.5\text{-}7)$$

Eq. (10.5-7) is quadratic in $z = \sqrt{E_y}$ so that we can write

$$az^2 + bz + c = 0 \qquad ...(10.5\text{-}8)$$

where $a = 1 + \dfrac{M_y}{M_Y}$, $b = -(2/M_Y)\sqrt{M_x M_y E_x}\cos\theta$

and $c = -E_x(1 - M_x/M_Y) - Q$

Eq. (10.5-8) has the solution $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...(10.5-9)

Written explicitly we then get

$$\sqrt{E_y} = \frac{1}{M_Y + M_y}\left\{(M_x M_y E_x)^{1/2}\cos\theta\right\} \pm \left[M_x M_y E_x \cos^2\theta + (M_Y + M_y)\left\{QM_Y + E_x(M_Y - M_x)\right\}\right]^{1/2} \qquad ...(10.5\text{-}10)$$

If we write $Q' = -Q$ then for endoergic reactions, $Q' > 0$ since $Q < 0$. In this case if $E_x = 0$, we have

$$b = 0 \quad \text{and} \quad c = -Q = Q' > 0$$

The solution for z in this case becomes

$$z = \sqrt{E_y} = \pm\frac{\sqrt{-4ac}}{2a} = \pm\sqrt{-Q'/a}$$

Since both a and Q' are positive $z = \sqrt{E_y}$ is imaginary in this case. This means that the reaction is not possible with $E_x = 0$. A minimum energy $E_x = E_{min}$ is needed to initiate endoergic reaction. In this case the term under the square root sign in Eq. (10.5-9) must be zero so that we get

$$b^2 - 4ac = 0$$

Substituting for a, b and c, we get

$$\frac{4}{M_Y^2}(M_x M_y E_{min})\cos^2\theta = 4\left(1 + \frac{M_y}{M_Y}\right)\left\{-Q - E_{min}\left(1 - \frac{M_x}{M_Y}\right)\right\}$$

which gives

$$E_{min} = -\frac{(M_y + M_Y)Q}{M_y + M_Y - M_x - (M_x M_y/M_Y)\sin^2\theta} \qquad ...(10.5\text{-}11)$$

Since $Q < 0$, $E_{min} > 0$.

Using Eq. (10.5-2) we get

$$E_{min} = \frac{-(M_y + M_Y)Q}{M_x - Q - (M_x M_y/M_Y)\sin^2\theta} \qquad ...(10.5\text{-}12)$$

E_{min} depends on the angle at which the particle y is emitted. When $\theta = 0$, *i.e.*, y is emitted in the forward direction, E_{min} has the lowest value and is known as the *threshold energy* for the endoergic reaction and is usually written as E_{th}. From Eq. (10.5-12) we get

$$E_{th} = -\frac{(M_y + M_Y)Q}{M_x - Q} \qquad ...(10.5\text{-}13)$$

Since $Q << M_x$, we can neglect it in the denominator of Eq. (11.5-13). Also we can replace $M_y + M_Y$ in the numerator by $M_x + M_X$. So we get finally

$$E_{th} \approx -Q.\frac{M_x + M_X}{M_X} = -Q\left(1 + \frac{M_x}{M_X}\right) \quad ...(10.5\text{-}14)$$

So by measuring the minimum energy E_{th} at which an exoergic reaction is initiated it is possible to determine the Q value of the reaction.

An inspection of Eq. (10.5-10) shows that under certain circumstances E_y will be a double-valued function of the projectile energy E_x *i.e.*, for a given E_x, there may be two values of E_y, the energy of the emitted particle. This happens only for endoergic reactions. The double valued nature of E_y is revealed in Fig. 10.6 for the $^3H(p, n)\,^3He$ endoergic reaction which has $Q = -0.7638$ MeV. Eq. (10.5-10) also shows that E_y is single valued if the following condition is satisfied :

$$QM_Y + E_x (M_Y - M_x) \geq 0$$

or,

$$E_x \geq \frac{-QM_Y}{M_Y - M_x}$$

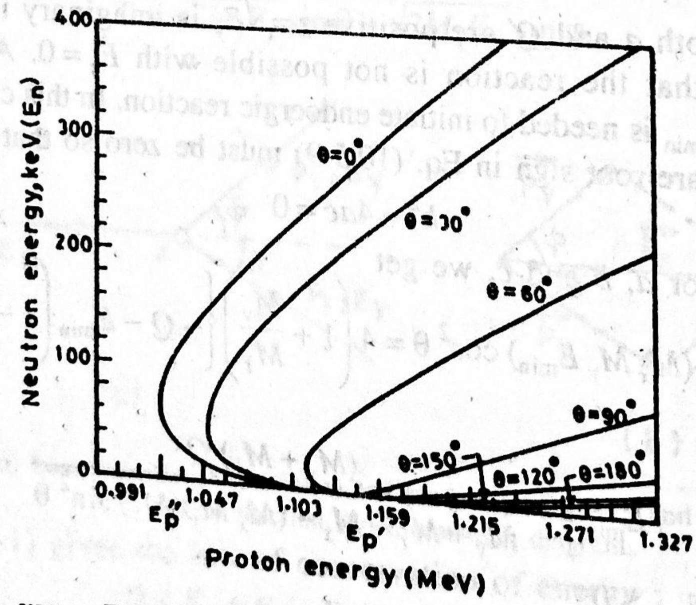

Fig. 10.6. E_n versus E_p graph in $^3H(p,n)\,^3He$ reaction. Double valued nature of neutron energy should be noted.

Thus there is a limiting energy of the projectile above which the emitted particle energy will be single valued. This is given by

$$E'_x = -\frac{QM_Y}{M_Y - M_x} \quad ...(10.5\text{-}15)$$

For the case cited above $E'_x = 1.145$ MeV. For projectile energy greater than E'_x, the product particle y can be emitted at all angles between 0° and a maximum angle θ_{max}, which can be found with the help of Eq. (10.5-10).

Exoergic reaction :

In this case, the reaction can occur for all values of E_x including $E_x = 0$. For $E_x = 0$, the incident momentum is zero and hence the sum of the momenta of the product particles must be zero : $\mathbf{p}_Y + \mathbf{p}_y = 0$. This means that Y and y proceed in opposite directions, so that $\theta + \varphi = \pi$. Also in this case $Q = E_y + E_Y$.

In general Eq. (10.5-7) gives for $Q > 0$ only one value of E_y, the energy of the emitted particle, for a given E_x and at a given angle of emission θ. All values of θ are possible. Hence there is an energy distribution of the emitted particles between a maximum at $\theta = 0$ and a minimum at $\theta = \pi$. In the solution for Eq. (10.5-7) given by Eq. (10.5-9) the plus sign has to be chosen if a positive value of the momentum p_y is to be obtained.

As we shall see in § 13.4 *c* exploitation of the rigid correlation between E_y and θ is the only way of obtaining monoenergetic neutron beams of different energies.

10.6 Experimental d...