6.10 Cross-section of nuclear reactions

All particles incident on a target do not produce a nuclear reaction, only a small fraction does. For a proper understanding of nuclear reactions, we are to known the probability that a single particle will interact with an individual target nucleus. In this context, an important parameter is the ***reaction cross-section***, σ. It is a *quantitative measure* of the *probability of occurrence of a nuclear reaction* and is a type of *effective* area presented by the target nucleus to that projectile for a specific reaction. It has nothing to do with the geometric cross-sectional area of the target nucleus.

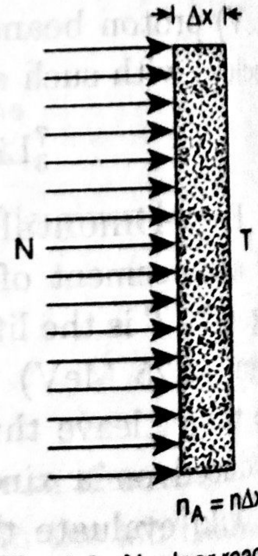

Fig. 6.9 Nuclear reaction cross-section: bombardment of target foil T

Let a parallel beam of N *monoenergetic* particles be incident per unit time normally on a target foil T of surface area A and thickness Δx, having n nuclei per unit volume. Now, the number ΔN of nuclei in the foil undergoing nuclear reaction will be proportional to (i) the *intensity of the beam* and (ii) the ***number of target nuclei*** contained in the foil (Fig. 6.9).

The intensity I of the beam $= N/A$ and the number of nuclei in the foil $= nA \cdot \Delta x$. So, the number of nuclei transmuted is

$$\Delta N \propto \frac{N}{A} nA \cdot \Delta x$$
$$= \sigma N n \cdot \Delta x$$
$$= \sigma N n_A$$

where σ = a constant, $n_A = n \cdot \Delta x$, the number of nuclei per unit area of target foil.

The constant σ is called the ***nuclear reaction cross-section*** and is given by

$$\therefore \quad \boxed{\sigma = \frac{\Delta N}{N n_A}} \qquad (6.10.1)$$

Since ΔN and N are pure numbers, σ has the dimensions of area and is therefore termed the ***cross-section.***

From (6.10.1) it is apparent that σ is the ***probability of occurrence of the reaction when a single particle*** ($N = 1$) ***falls on a single target nucleus*** ($n_A = 1$) ***present per unit area.***

Since the nuclear radius is $\sim 10^{-14}$ to 10^{-15} m, the cross-section is $\sim 10^{-28}$ m^2. The common unit of reaction cross-section is a ***barn*** being defined as :

$$\boxed{\textbf{1 barn} = \mathbf{10^{-28}\ m^2}}$$

Geometrical significance of σ — The geometrical significance of σ can be understood in reference to Fig. 6.10. If R be the effective radius of the target nucleus for a given reaction, then the projection of its surface on a plane normal to the direction of incidence of the incoming particles (assumed to be mass-points) is πR^2.

$\therefore$ Number of particles encountering each target nucleus $= \pi R^2 N_A$, where $N_A = N/A$ is the number of particles incident per unit area of the target.

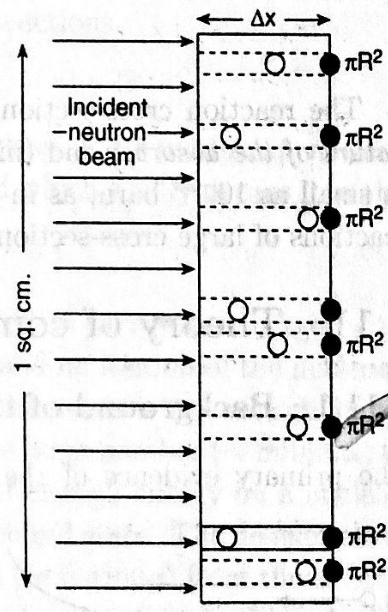

Fig. 6.10 Geometrical significance of nuclear reaction cross-section

But the number of nuclei per unit area of the target is n_A. So the number of incident particles intercepted by the target nuclei in the foil is :

$$n_A A \times \pi R^2 N_A = \pi R^2 N n_A \quad (\because N_A = N/A)$$

$\therefore$ The probability of encounter of a single particle ($N = 1$) with a single target nucleus per unit area ($n_A = 1$), that is, σ is given by

$$\sigma = \frac{\pi R^2 N n_A}{n_A} = \pi R^2$$

which is the ***geometrical significance*** of the reaction cross-section, σ.

Geometrically, therefore, the *reaction cross-section* is the *area of cross-section of an imaginary disc associated with each target nucleus such that if the incident projectile passes through it, the reaction occurs.*

Differential cross-section — In many nuclear reactions, the product particles are *not* emitted *isotropically* in all directions. The angular distribution is measured in terms of *differential cross-section, $d\sigma/d\Omega$*, the number of product particles emitted per sec in a solid angle $d\Omega$ at some angle θ relative to the direction of incident beam.

But, $$\frac{d\sigma}{d\Omega} = \frac{dN}{Nn_A} \Big/ d\Omega = \frac{dN/d\Omega}{Nn \cdot \Delta x} \tag{6.10.2}$$

where dN number of product nuclei are emitted per sec. in a *small* solid angle $d\Omega$ at θ with the incident beam, n being the number of nuclei per unit volume of target of thickness Δx.

The differential cross-section, $d\sigma/d\Omega$, however varies with the solid angle.

Total cross-section — The *total* or *integral cross-section*, σ, over solid angle Ω is

$$\sigma = \int_\Omega \frac{d\sigma}{d\Omega} d\Omega \tag{6.10.3}$$

The reaction cross-section, σ depends on (i) the *nature of the projectile*, (ii) the *nature of the absorber* and (iii) the *energy of the projectile*. It may vary from a value as small as 10^{-20} barn, as in neutrino interactions, to as high as 10^5 barn in nuclear reactions of large cross-section.