the diffraction effect parallel to vertical y-axis becomes negligible and diffraction will occur only in horizontal direction. In that case

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

where $I_0 = (CAa)^2$. This is the well-known expression for single slit diffraction pattern.

12.16 CONCAVE GRATING :

A concave reflection grating, as developed by Rowland, consists of a concave polished metal surface having very fine lines ruled on it. It diffracts the incident beam of light and at the same time focuses it without the use of additional lenses as in case of plane diffraction grating. Thus the spectra obtained here are free from chromatic and other aberrations. This grating can be used to investigate ultra-violet light for which glass lenses are not transparent. The most important advantage of this concave grating over plane grating is that it can produce a truly normal spectrum in which distance between two lines is proportional to the wavelength difference.

Theory of concave grating :

Let L and M be the corresponding points on the two consecutive polished strips of the concave grating GG_1 whose centre of curvature is at C (Fig. 12.16-1). Let a be the breadth of a ruling and b be the breadth of a polished strip. Then the distance between L and M will be $a+b$. Light rays of wavelength λ, starting from an illuminated vertical slit S, are incident on the corresponding points M and L along SM and SL making angles of incidence i and $(i + di)$ with the normals MC and LC respectively. Diffracted rays MP and LP from M and L are diffracted at angles θ and $(\theta + d\theta)$ respectively and converge at P. The condition of brightness or darkness at P will be determined by the path difference of P from S for the phase change of π produced at each reflection at M and L cancel each other. Path difference of P from S is given by,

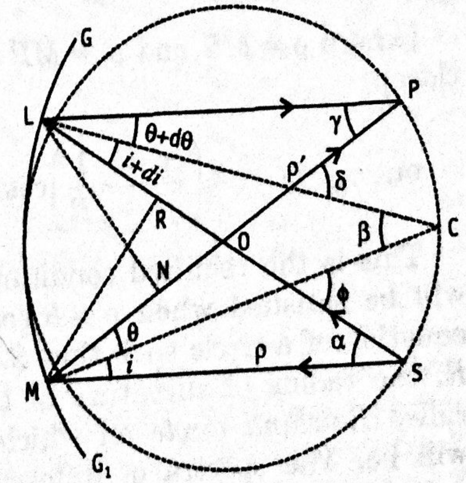

Fig.12.16-1

$$l = (SL + LP) - (SM + MP) = (SL - SM) - (MP - LP) \quad \text{...(12.16-1)}$$

With S as centre and SM as radius arc MR is drawn which is practically a straight line due to close proximity of M and L. Hence by geometry $\angle LMR = i$. Again with P as centre and PL as radius an arc LN is drawn which is practically a straight line. By geometry $\angle MLN = \theta$. Hence from Fig. 12.16-1.

$$(SL - SM) = LR = LM \sin i = (a + b) \sin i$$

and
$$(MP - LP) = MN = LM \sin\theta = (a + b) \sin\theta$$

Hence Eq. (12.16-1) reduces to,

$$l = (a + b)(\sin i - \sin\theta) \qquad ...(12.16\text{-}2)$$

If the points P and S are on the same side of C then

$$l = (a + b)(\sin i + \sin\theta) \qquad ...(12.16\text{-}3)$$

For maximum at P, $(a + b)(\sin i \pm \sin\theta) = m\lambda$ ...(12.16-4)

This condition for maxima will be true for any pair of diffracted rays of a given wavelength from the corresponding points, for a given order number m. Hence on differentiating Eq. (12.16-4) we get,

$\cos i\, di \pm \cos\theta\, d\theta = 0$ [$\because$ $(a + b)$ and $m\lambda$ are constants].

From Fig. 12.16-1, $\phi = i + \alpha = i + di + \beta$; or, $di = \alpha - \beta$

$\delta = \beta + \theta = \theta + d\theta + \gamma$; or, $d\theta = \beta - \gamma$.

$\therefore$ $(\alpha - \beta)\cos i - (\beta - \gamma)\cos\theta = 0$ [considering the case when P and S are on the opposite side of C].

or,
$$\left(\frac{LM\cos i}{\rho} - \frac{LM}{R}\right)\cos i - \left(\frac{LM}{R} - \frac{LM\cos\theta}{\rho'}\right)\cos\theta = 0$$

[where $\rho = MS$ and $\rho' = MP = LP$ (practically) for L and M are very close].

or,
$$\left(\frac{\cos i}{\rho} - \frac{1}{R}\right)\cos i - \left(\frac{1}{R} - \frac{\cos\theta}{\rho'}\right)\cos\theta = 0 \qquad ...(12.16\text{-}5)$$

This is the required condition for maximum at P and this equation will be satisfied when $\rho = R\cos i$ and $\rho' = R\cos\theta$. This is the polar equation of a circle such that S and P may lie on the circle of diameter R, the radius of curvature of the concave grating GG_1. This circle is called *Rowland circle* on which the source S and the spectrum at P will lie. The spectra of different orders will lie on the circle.

Mounting of concave grating :

(a) Rowland Mounting:

The property of Rowland circle was used by Rowland to mount the concave grating in such a way, that the spectrum can be photographed when the angle of diffraction θ is zero. Grating L and the photographic plate P are fixed on an arm at a distance LP which is equal to the radius of curvature of the concave grating (Fig. 12.16-2). The ends of the arm are constrained to move along two girders SL and SP at right angles to each other so that for all positions of LP the corner S is on the Rowland circle of which LP is the diameter. The illuminated vertical slit is placed at S. The diffracted rays from the grating cross at P, the centre of curvature of the grating and hence the angle of diffraction θ is zero. The angle of incidence is $\angle SLP = i$.

Hence,
$$(a + b) \sin i = m\lambda$$
$[\because \theta = 0;$ from Eq. (12.16-4)].

or,
$$(a + b)\frac{SP}{PL} = m\lambda$$

Now $(a + b)$, PL and m are constants and hence SP is proportional to λ. Thus in a given order m of the spectrum, lights of different wavelengths are arranged on SP so that SP is proportional to λ. This type of spectrum is called *Normal Spectrum.*

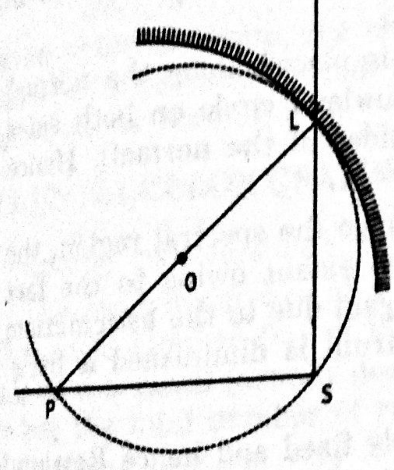

Fig. 12.16-2

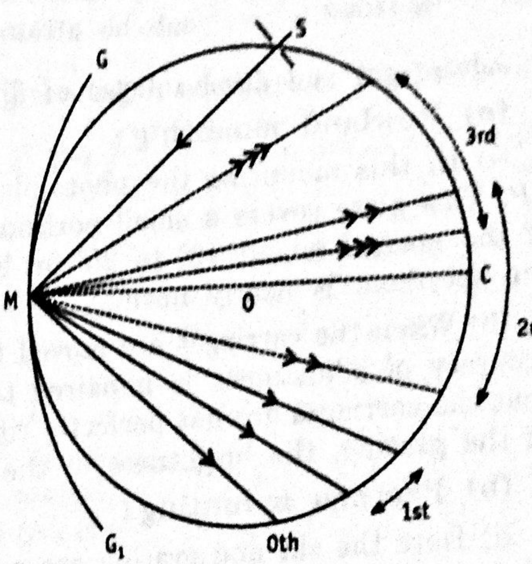

Fig. 12.16-3

(b) Paschen's mounting:

In this mounting, the grating GMG_1 and the slit S are fixed while the photographic plate is curved to make it suitable for any convenient position of Rowland circle. The plate is mounted in a circular steel track (not shown in Fig. 12.16-3). A typical mounting is shown in Fig. 12.16-3 and the approximate positions of visible spectra of different orders are shown. Usually the grating and the slit are mounted on separate piers, while the mounting of the plate is extremely rigid. The spectrum obtained would be nearly normal when the angle of diffraction is small.

(c) Eagle mounting:

Here the grating (G) and the photographic plate (P) are mounted on the Rowland circle (C) (Fig. 12.16-4). The plate (P) is curved to fit the Rowland circle (C). When the grating is moved and rotated to the G_1 position to suit the Rowland circle C_1, the photo-plate is also rotated to the P_1 position to suit the Rowland circle C_1. The slit S is mounted by the side of or just above or below the plate, so that the virtual image of S, formed by total reflection in the prism R, is at the centre of the plate P. By this, the property of Rowland circle is also utilised by the slit. In this arrangement $\theta = i$ and the reflected rays from the prism R goes to the grating which gives diffracted rays. Compactness of its mounting makes it highly suitable in vacuum spectrograph. As the space is small, the temperature can be easily controlled and vacuum can be attained quickly.

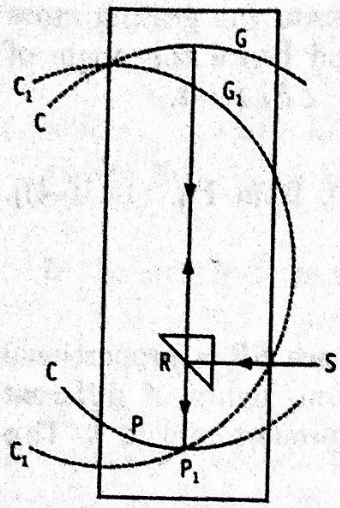

Fig. 12.16-4

Advantages and disadvantages of different mountings :

(a) Rowland mounting :

(i) In this mounting the photo-plate P is placed along the normal LP. This plate covers a small portion of Rowland circle on both sides of the normal (about 10° to 20° on both sides of the normal). Hence the spectrum is nearly normal.

(ii) When the carriages are moved to change the spectral region, the accuracy of adjustment is impaired to some extent owing to the fact that the carriages are not perfectly rigid. Again due to the astigmatism of the grating, the brightness of the spectrum is diminished a little.

(b) Paschen mounting :

(i) Here the slit and grating are relatively fixed and hence Rowland circle is accurately located for a given problem. Different orders of the spectrum are simultaneously focussed on the photo-plate bent to suit the Rowland circle and fixed tightly to the rim of a steel tube.

(ii) Normal dispersion is obtained near about the normal to the grating while on both sides of the normal, dispersion increases. A large space is necessary for this type of mounting.

(c) Eagle mounting :

(i) Mounting can be made within a small space which can be quickly evacuated and whose temperature can be kept constant at a desired value. This grating is suitable for studying ultra-violet light by making the mounting space vacuum. Due to less astigmatism the brightness of the spectrum is greater. Higher orders can be obtained than in the case of Rowland mounting.

(ii) To pass from one spectral region to another the grating is to be rotated and adjustments are to be made for focussing.

Dispersion :

If the light from the source at S contains waves of different lengths then θ will be different for light of different wavelengths though i will remain the same for all wavelengths. Hence differentiating Eq. (12.16-4) we get,

$$d \cos \theta \, d\theta = m d\lambda. \qquad \text{...(12.16-6)}$$

If $d\lambda$ covers an arc ds of Rowland circle then $d\theta = ds/\rho$. Hence, $d \cos \theta . ds/\rho = m d\lambda$;

or,
$$\frac{ds}{d\lambda} = \frac{\rho m}{d \cos \theta} \qquad \text{...(12.16-7)}$$

If the diffracted rays proceed near to the normal (MC) then $\cos \theta = 1$.

Hence $\frac{ds}{d\lambda} = \frac{\rho m}{d} =$ constant for a given order number m. Thus in the given order spectrum, the change in the length of the arc of Rowland circle will be proportional to the wavelength. This shows that the spectrum is *normal*.